LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2014

MT 3503 - VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date : 31/10/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART – A

Answer ALL questions: **(10 × 2 = 20)**

1. If $\phi (x,y,z) = x^2y + y^2x + z^2$, find $\nabla\phi$ at (1,1,1).
2. Show that the vector $3x^2y\,\bar{i} - 4xy^2\,\bar{j} + 2xyz\,\bar{k}$ is solenoidal.
3. If $\bar{F} = x^2\bar{i} + y^2\bar{j}$, evaluate $\int \bar{F}.d\bar{r}$ along the line y = x from (0,0) to (1,1).
4. Show that $\bar{F} = x^2\bar{i} + y^2\,\bar{j} + z^2\bar{k}$ is a conservative vector field.
5. State Green's theorem.
6. Show that $\iint_S Curl\,\bar{F}\,.\bar{n}\,ds = 0$ where S is any closed surface.
7. Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.
8. Solve $y = (x-a)p - p^2$.
9. Solve $(D^2 + 4)y = 0$.
10. Find the complimentary function for the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = 10 + \frac{5}{x^2}$.

PART – B

Answer any FIVE questions: **(5 × [illegible] = 40)**

11. Find the directional derivative of $\phi = xy + yz + zx$ in the direction of the vector $\bar{i} + \bar{2j} +$ [illegible] $\bar{k}$ at (1,2,0).
12. If $\bar{F} = x^2y\bar{i} + y^2z\bar{j} + z^2x\bar{k}$, find Curl Curl $\bar{F}$.
13. Evaluate $\square_S\ \bar{F}\,.\bar{n}\,ds$ where $\bar{F} = yz\bar{i} + zx\bar{j} + xy\,\bar{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.
14. Evaluate by Stokes theorem $\int_C e^x\,dx + 2y\,dy - dz$ where C is the curve $x^2 + y^2 = 4$; z = 2.
15. Solve $y(1 - p^2) = 2px$.
16. Solve $x = p^2 + y$.
17. Solve $(D^2 + 5D + 4)\,y = x^2 + 7x + 9$.
18. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 5y = \mathrm{Cos}(\log x)$.

PART – C

Answer any TWO questions: **($2 \times 20 = 40$)**

19. (a) Find the value of the constant a,b,c so that the vector $\bar{F} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational.

(b) Find the unit normal to the surface $x^2 y + 2xz^2 = 8$ at (1,0,2).

(c) If $F = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20xz^2\bar{k}$, evaluate $\int_C \bar{F}.d\bar{r}$ where C is the straight line joining (0,0,0) to (1,1,1). (6-6+8)

20. Verify Green's theorem in the XY plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region given by x = 0; y = 0 ; x + y = 1.

21. (a) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

(b) Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x \cos^2 x}{y^2}$.

(c) Solve $2p^2 - (x+2y^2)p + xy^2 = 0$. (6+7+7)

22. (a) Solve $(D^2 + 4D + 3)y = e^x \sin x + xe^{3x}$.

(b) Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$. (10 + 10)

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